Sector improved residue subtraction

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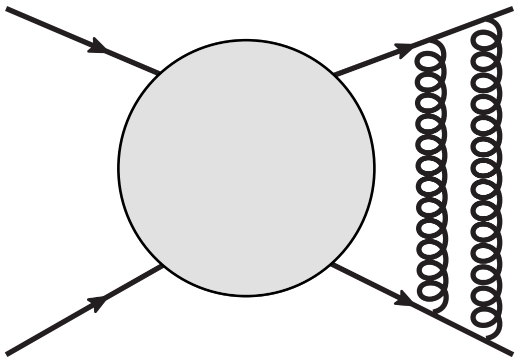
STRIPPER (NNLO - Subtraction)

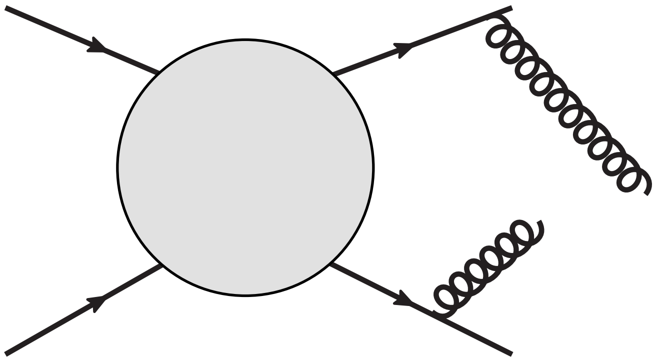
Sec**T**o**R** **I**mproved **P**hase s**P**ac**E** for real **R**adiation

Introduced: 2010 [Czakon, 2010]

- Scattering
  - Top Quark Pair Production [Czakon; 2011]
  - $gg \to H + jet$ [Boughezal, Caola, Melnikov, Petriello,Schulze; 2013]
- Decays
  - Charmless Bottom Decay [Brucherseifer, Caola, Melnikov; 2013]
  - Top Decay [Brucherseifer, Caola, Melnikov; 2013]
  - $Z \to e\ e$ [Boughezal, Melnikov, Petriello; 2011]
  - Muon Decay [Caola, Czarnecki, Liang, Melnikov, Szafron; 2014]

CDR $\leftrightarrow$ 't Hooft Veltman scheme (HV)





- Dimension of Polarization Vector and Momentum

	CDR	't Hooft Veltman
Resolved Particle	d - dimensions	4 - dimensions
Unresolved Particle	d - dimensions	d - dimensions

- Problem in CDR
  - Multiplicity increases number of parameterized dimensions in the phase space
  - ϵ – dependence of matrix elements

“Analytic” subtraction schemes

$$\mathrm{d}\sigma^{RR,\mathrm{sub}} = \int_{n+2} \left(\mathrm{d}\sigma^{RR} - \mathrm{d}\sigma^{RR,\mathrm{single}} - \mathrm{d}\sigma^{RR,\mathrm{double}}\right)$$

$$\mathrm{d}\sigma^{RV,\mathrm{sub}} = \int_{n+1} \left(\left[\mathrm{d}\sigma^{RV} + \int_1 \mathrm{d}\sigma^{RR,\mathrm{single}}\right] - \mathrm{d}\sigma^{RV,\mathrm{single}}\right)$$

$$\mathrm{d}\sigma^{VV,\mathrm{fin}} = \int_{n} \left(\mathrm{d}\sigma^{VV} + \int_1 \mathrm{d}\sigma^{RV,\mathrm{single}} + \int_2 \mathrm{d}\sigma^{RR,\mathrm{double}}\right)$$

- NLO
  - Dipole Subtraction, FKS — [Catani, Seymour; 1996] [Frixione, Kunszt, Signer; 1996]
- NNLO
  - Antenna – Subtraction — [Gehrmann, Glover et al; Weinzierl; 2005]
  - Somogyi, Trócsányi, Del Duca — [Somogyi, Trócsányi, Del Duca; 2005]
- Poles cancel analytically
  - $\rightarrow$ Phase – Space integrations are 4 dimensional

Subtraction for RR (STRIPPER)

- Double Real Radiation

$$\mathrm{d}\sigma^{RR} = \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}^{(0)}_{n+2} \middle| \mathcal{M}^{(0)}_{n+2} \right\rangle$$

- Concept

1. Use Selector Functions to split phase space in Triple - and Double Collinear Sectors
2. Use a physical parametrization (angles, energy)
3. Physical Sector Decomposition: Factorization of non-commuting singularities
4. Generate Subtraction Terms using + - distribution
5. Laurent series in ϵ
   - Coefficients are integrated numerically

Subtraction for RR (STRIPPER)

- Selector Functions

$$1 = \sum \sum \left[S_{\mathrm{TC}} + \sum S_{\mathrm{DC}} \right] \Rightarrow \boxed{\sigma^{RR} = \sum_S \sigma_S^{RR}}$$

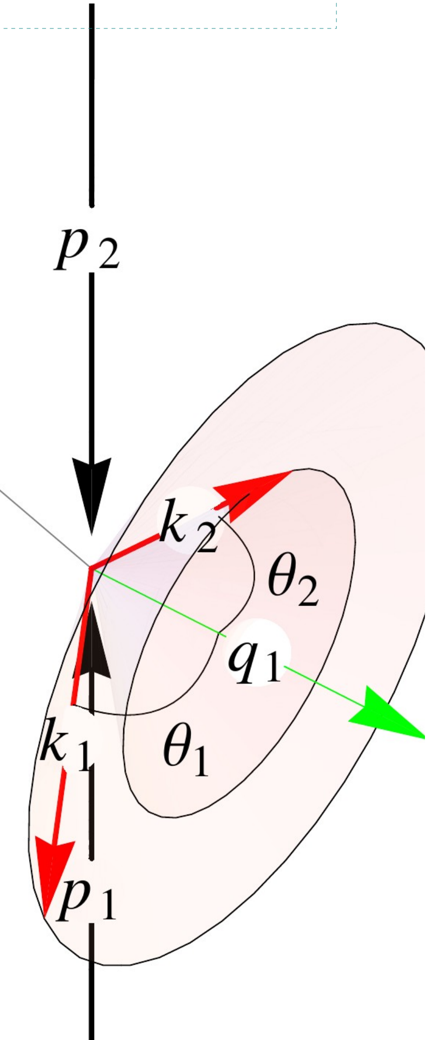
  - Triple Collinear Sector
    - 3 specific partons can become collinear and 2 of them soft
  - Double Collinear Sector
    - 2 specific pairs of partons can become collinear and 2 of them soft
  - Remark: Used also in RV and NLO (FKS - Subtraction)

Physical Parametrization (STRIPPER)

- Triple Collinear Sector Final (Example)

$$q_1^\mu \equiv q_1^0 \begin{pmatrix} 1 \\ \hat{\boldsymbol{q}}_1 \end{pmatrix} \ , \quad k_1^\mu \equiv k_1^0 \begin{pmatrix} 1 \\ \hat{\boldsymbol{k}}_1 \end{pmatrix} \ , \quad k_2^\mu \equiv k_2^0 \begin{pmatrix} 1 \\ \hat{\boldsymbol{k}}_2 \end{pmatrix}$$

$$\begin{aligned} \hat{\boldsymbol{q}}_1 &\equiv \hat{\boldsymbol{n}}^{(d-1)}(\alpha_1, \alpha_2, \dots) \ , \\ \hat{\boldsymbol{k}}_1 &\equiv \boldsymbol{R}_1^{(d-1)}(\alpha_1, \alpha_2, \dots)\hat{\boldsymbol{n}}^{(d-1)}(\theta_1, \phi_1, \rho_1, \rho_2, \dots) \ , \\ \hat{\boldsymbol{k}}_2 &\equiv \boldsymbol{R}_1^{(d-1)}(\alpha_1, \alpha_2, \dots)\boldsymbol{R}_2^{(d-1)}(\phi_1, \rho_1, \rho_2, \dots)\hat{\boldsymbol{n}}^{(d-1)}(\theta_2, \phi_2, \sigma_1, \sigma_2, \dots) \end{aligned}$$



  - Collinear limits are parameterized easily

$$\begin{aligned} \hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{q}}_1 &= \cos\theta_1 = 1 - 2\eta_1 \\ \hat{\boldsymbol{k}}_2 \cdot \hat{\boldsymbol{q}}_1 &= \cos\theta_2 = 1 - 2\eta_2 \\ \hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2 &= \cos\theta_1 \cos\theta_2 + \cos\phi_2 \sin\theta_1 \sin\theta_2 \end{aligned}$$

$$\phi_2 \equiv \phi_2(\theta_1, \theta_2, \zeta)$$

$$\phi_2(\theta_1, \theta_1, \zeta) = 0$$

  - Variables that appear in arbitrary scalar products
    - All other particles in 4 D (HV)

$$\theta_1, \phi_1, \rho_1, \\ \theta_2, \phi_2, \sigma_1, \sigma_2$$

Physical Sector Decomposition → Factorization

- Double – Soft Limit (schematic)

$$S = \int_0^1 \mathrm{d}\hat{\xi}_1 \mathrm{d}\hat{\xi}_2 \, \frac{\hat{\xi}_1^{-2\epsilon}\hat{\xi}_2^{-2\epsilon}}{\left(\hat{\xi}_1 + \hat{\xi}_2\right)^2}$$

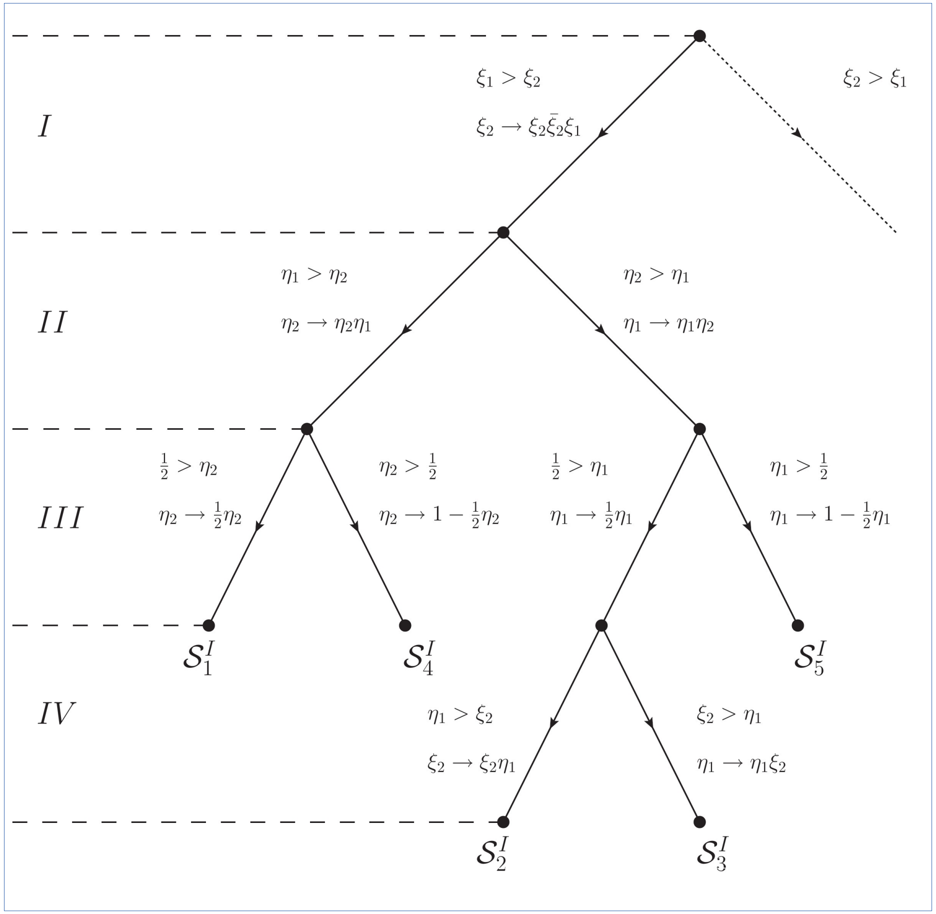
impose order of limits

$$1 = \theta\left(\hat{\xi}_1 > \hat{\xi}_2\right) + \theta\left(\hat{\xi}_2 > \hat{\xi}_1\right)$$

$$S_{1>2} = \int_0^1 \mathrm{d}\xi_1 \mathrm{d}\xi_2 \, \frac{\xi_1^{-1-4\epsilon}\xi_2^{-2\epsilon}}{(1+\xi_2)^2}$$

Rescaled parameters: $\hat{\xi}_2 \to \xi_1\xi_2 \quad \hat{\xi}_1 \to \xi_1$

- Triple – Collinear Sector
  - Additional Splitting into 5 angular sectors



$$\boxed{\sigma^{RR} = \sum_S \sigma_S^{RR}}$$

Subtraction Terms (STRIPPER)

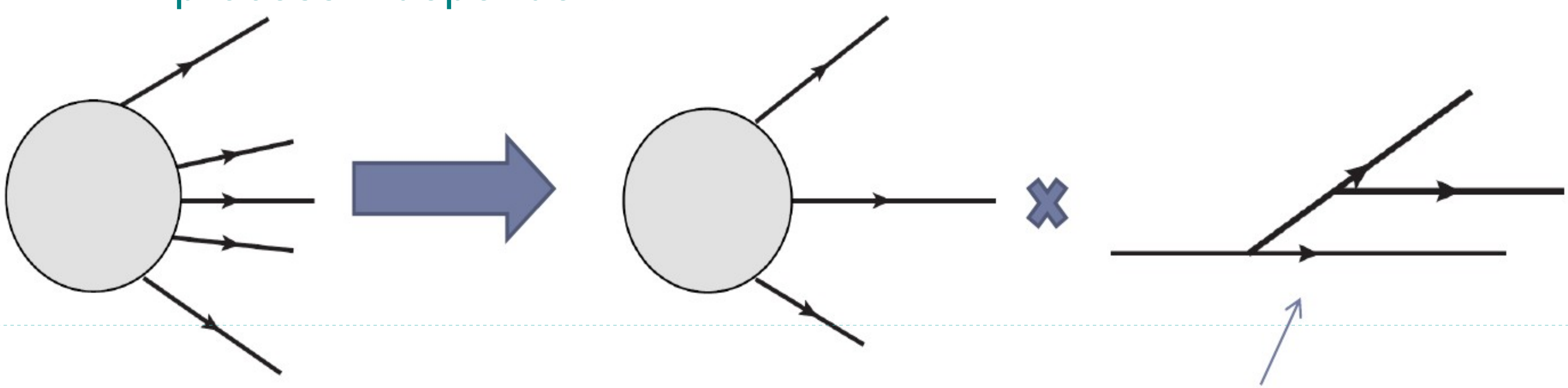
$$\boxed{\sigma^{RR} = \sum_S \sigma_S^{RR}}$$

$$\sigma_S^{RR} = \int \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\eta_1 \mathrm{d}\eta_2 \frac{F_S\left(\xi_1, \xi_2, \eta_1, \eta_2\right)}{\xi_1^{1-b_1\epsilon}\xi_2^{1-b_2\epsilon}\eta_1^{1-b_3\epsilon}\eta_2^{1-b_4\epsilon}}$$

- Generating Subtraction Terms

$$\int_0^1 \mathrm{d}x \, \frac{f\left(x\right)}{x^{1-b\epsilon}} = \frac{\boxed{f\left(0\right)}}{b\epsilon} + \int_0^1 \mathrm{d}x \, \frac{f\left(x\right) - \boxed{f\left(0\right)}}{x^{1-b\epsilon}}$$

- Use known IR - limits of QCD amplitudes
  - process independent



IR – Structure $\rightarrow$ Single Unresolved (SU)

- NLO like pole cancellation ~ $\left|\mathcal{M}^{(0)}_{n+1}\right|$
  - Real – Virtual (including subtraction terms)

$$\mathrm{d}\sigma^{RV} = \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re}\left(\boxed{\left\langle \mathcal{M}^{(0)}_{n+1}\middle| \mathbf{Z}^{(1)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1}\right\rangle} + \left\langle \mathcal{M}^{(0)}_{n+1}\middle| \mathcal{F}^{(1)}_{n+1}\right\rangle\right)$$

  - Collinear Factorization Contribution
  - Integrated single subtraction terms Double Real
    - Subtraction terms are not the same as in Real – Virtual and Factorization
      - Sector decomposition → different parametrization $\hat{\xi}_2 \rightarrow \xi_1\xi_2$

1. Different Range of Integration
2. Subtraction terms are not minimal in physical variables

STRIPPER in 't Hooft Veltman

- Idea: Force SU contribution to be finite
  - Allows to set (using "Jet Function")
    - Momenta of (n+1) resolved particles to 4d
    - Spin dof to 4d → Matrix Elements in 4d

$$F_J \cdot \prod_{i=1}^{n} \delta^{(-2\epsilon)}(q_i)$$

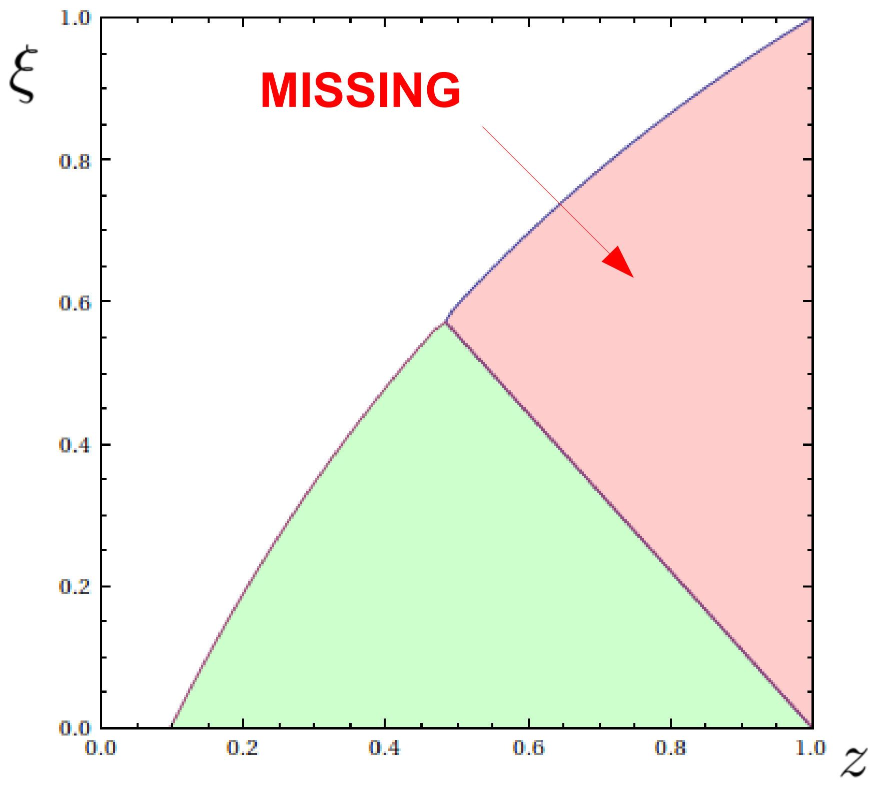
- Realization in RR:
  - Azimuthal average
    - Problem in final collinear (Triple Collinear Sector) $\phi_2 \equiv \phi_2(\theta_1, \theta_2, \zeta)$
    - Replace pole splitting functions by averaged ones
    - Use iterated collinear limits
  - Correct subtraction terms and integration region

Correct missing parts in RR

- Example Case: Initial – state collinear
  - Identify pole and subtraction (collinear pole, soft subtraction)
  - Compare integration range
  - Add missing piece

$$\frac{1}{\epsilon}\int\frac{dz}{z}\,P\left(z\right)\mathrm{d}\sigma^{(1)}_{\mathrm{NLO}}\left(zp_1,p_2\right)$$

Soft Subtraction Term
Variable ξ



Double Unresolved (DU)

- 1 Loop Finite Remainder Part is finite [Weinzierl, 2011]
- SU is finite

Double Unresolved Part is finite ~ $\left|\mathcal{M}_n^{(0)}\right|$

- Allows to set (using "Jet Function")
  - Momenta of n resolved particles to 4d
  - Spin dof to 4d $\longrightarrow$ Matrix Elements in 4d
- Use Averaged Splitting functions

$$F_J \cdot \prod_{i=1}^{n-1} \delta^{(-2\epsilon)}(q_i)$$

Formulation of STRIPPER in 4 dimensions

Example: gg → tt +X

- SU – contribution

$$\sigma = \frac{\alpha_s^4}{m_t^2}\,\hat{\sigma} \qquad \beta = \sqrt{1 - \frac{4m_t^2}{s}} = 0.5$$

	CDR	't Hooft Veltman	Agreement (CDR - HV)
$1/\varepsilon^2$	$(-8.1 \pm 7.5) \cdot 10^{-6}$	$(1.1 \pm 0.8) \cdot 10^{-5}$	
$1/\varepsilon$	$(-5.7 \pm 5.9) \cdot 10^{-5}$	$(4.2 \pm 4.1) \cdot 10^{-5}$	
Finite Term	(0.2580 ± 0.0003)	(0.2584 ± 0.0002)	(-0.0004 ± 0.0004)

- DU -contribution

	CDR	't Hooft Veltman	Agreement (CDR -HV)
$1/\varepsilon^4$	$(-1.6 \pm 0.9) \cdot 10^{-6}$	$(-8.6 \pm 8.9) \cdot 10^{-7}$	
$1/\varepsilon^3$	$(-5.2 \pm 6.1) \cdot 10^{-6}$	$(3.2 \pm 5.2) \cdot 10^{-6}$	
$1/\varepsilon^2$	$(1.3 \pm 2.4) \cdot 10^{-5}$	$(-1.0 \pm 1.7) \cdot 10^{-5}$	
$1/\varepsilon$	$(7.4 \pm 9.3) \cdot 10^{-5}$	$(1.9 \pm 6.2) \cdot 10^{-5}$	
Finite Term	(-0.03041 ± 0.00035)	(-0.03045 ± 0.00042)	(0.00004 ± 0.00042)

Summary

- Idea
  - Separate Finite Remainders
  - Force SU and DU to be finite separately
  - Impose “Jet-Function” to constrain resolved particles to 4d
- Conclusion
  - Generalization of STRIPPER to arbitrary multiplicities
  - All resolved particles in 4 dimensions
  - All matrix elements in 4 dimensions
- Outlook
  - Long-Term Goal: Publicly available software
    - 1-Loop and 2-Loop Finite Remainders provided by user
  - Many technical improvements

- Detailed description currently being written down

Back Up - Slides

Separate contributions (DU)

[CDR]

	$1/\epsilon^4$	$1/\epsilon^3$	$1/\epsilon^2$	$1/\epsilon$	ϵ^0
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{VV}}$	0.0321959	0.135003	0.177418	0.04517	−0.1242
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{RV}}$	−0.0724423(9)	−0.456495(4)	−1.196150(11)	−1.81962(4)	−2.8562(1)
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{RR}}$	0.0402448(2)	0.321486(1)	1.045064(6)	1.61821(4)	1.3065(3)
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{F1}}$		−0.154649(4)	−0.447655(20)	0.09385(8)	1.8313(2)
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{F2}}$		0.154650	0.421336	0.06247	−0.1878
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{CDR}}$	−0.0000016(9)	−0.000005(6)	0.000013(24)	0.00007(9)	−0.0304(4)

Table 1: Double-unresolved (DU) contributions to the partonic cross section $gg \to t\bar{t} + X$, with X consisting of up to two gluons, evaluated in conventional dimensional regularization (CDR). The error estimates quoted in parentheses are due to Monte Carlo integration. The definition of partial contributions is given in the text.

[HV]

	$1/\epsilon^4$	$1/\epsilon^3$	$1/\epsilon^2$	$1/\epsilon$	ϵ^0
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{VV}}$	0.0321959	0.086177	0.021985	−0.03200	0
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{RV}}$	−0.0724415(9)	−0.346630(3)	−0.702124(8)	−1.04640(3)	−2.39100(8)
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{RR}}$	0.0402447(2)	0.260452(1)	0.706469(6)	1.06119(3)	1.8461(2)
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{F1}}$		−0.154646(4)	−0.283008(15)	0.08326(5)	0.5144(1)
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{F2}}$		0.154650	0.256668	−0.06603	0
$\tilde{\sigma}_{\mathrm{DU}}^{\mathrm{HV}}$	−0.0000009(9)	0.000003(6)	−0.000010(17)	0.00002(6)	−0.0304(2)

Table 2: Double-unresolved (DU) contributions to the partonic cross section $gg \to t\bar{t} + X$, with X consisting of up to two gluons, evaluated in 't Hooft-Veltman regularization (HV). The error estimates quoted in parentheses are due to Monte Carlo integration. The definition of partial contributions is given in the text.

Separate contributions (SU)

[CDR]

	$1/\epsilon^2$	$1/\epsilon$	ϵ^0
$\tilde{\sigma}^{\mathrm{RR}}_{\mathrm{SU}}$	0.064772(4)	0.42742(3)	1.0623(3)
$\tilde{\sigma}^{\mathrm{RV}}_{\mathrm{SU}}$	−0.064780(6)	−0.31419(4)	−0.6044(2)
$\tilde{\sigma}^{\mathrm{F}}_{\mathrm{SU}}$		−0.11329(3)	−0.1999(1)
$\tilde{\sigma}^{\mathrm{A}}_{\mathrm{SU}}$			−0.00737(2)
$\tilde{\sigma}^{\mathrm{CDR}}_{\mathrm{SU}}$	−0.000008(8)	−0.00006(6)	0.2506(3)

Table 3: Single-unresolved (SU) contributions to the partonic cross section $gg \to t\bar{t} + X$, with X consisting of up to two gluons, evaluated in conventional dimensional regularization (CDR). The error estimates quoted in parentheses are due to Monte Carlo integration. The definition of partial contributions is given in the text.

[HV]

	$1/\epsilon^2$	$1/\epsilon$	ϵ^0
$\tilde{\sigma}^{\mathrm{RR}}_{\mathrm{SU}}$	0.064780(5)	0.25429(3)	0.2584(2)
$\tilde{\sigma}^{\mathrm{RV}}_{\mathrm{SU}}$	−0.064770(7)	−0.14096(2)	0
$\tilde{\sigma}^{\mathrm{F}}_{\mathrm{SU}}$		−0.11329(2)	0
$\tilde{\sigma}^{\mathrm{A}}_{\mathrm{SU}}$			−0.00734(1)
$\tilde{\sigma}^{\mathrm{HV}}_{\mathrm{SU}}$	0.000011(8)	0.00004(4)	0.2511(2)

Table 4: Single-unresolved (SU) contributions to the partonic cross section $gg \to t\bar{t} + X$, with X consisting of up to two gluons, evaluated in 't Hooft-Veltman regularization (HV). The error estimates quoted in parentheses are due to Monte Carlo integration. The definition of partial contributions is given in the text.